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DESIGN AND MODELING THE Φ-F CHART METHOD FOR ACTIVE SOLAR ENERGY SYSTEMS

Authors: *S. Poles, M. Venturin*

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Abstract: The aim of this paper is to show how to implement the φ-f chart method with Scilab. This model can be used to calculate the solar fraction for hot water production with respect to the global request. This Scilab code can be used to estimate the thermal performance of solar energy systems for domestic water, industrial process heating, and space heating systems where the thermodynamics cycle efficiency is independent of the heat supply temperature.

Contacts info@openeering.com

1. Introduction

In this paper we describe and model the φ-f chart method with Scilab. The φ-f chart method is a simulation technique used in solar energy systems for heating and cooling. This method is particularly useful for sizing system components. The difficulty of sizing system's components lies in the fact that these systems are influenced both by predictable data (e.g. collectors, storage tanks, …) and unpredictable data such as weather data.

The φ-f chart method is applied to the closed loop solar energy system as reported in Figure 1. This method is particularly useful in the simulation of absorption refrigerators, industrial process heating, and space heating systems where the thermodynamics cycle efficiency is independent of the heat supply temperature.

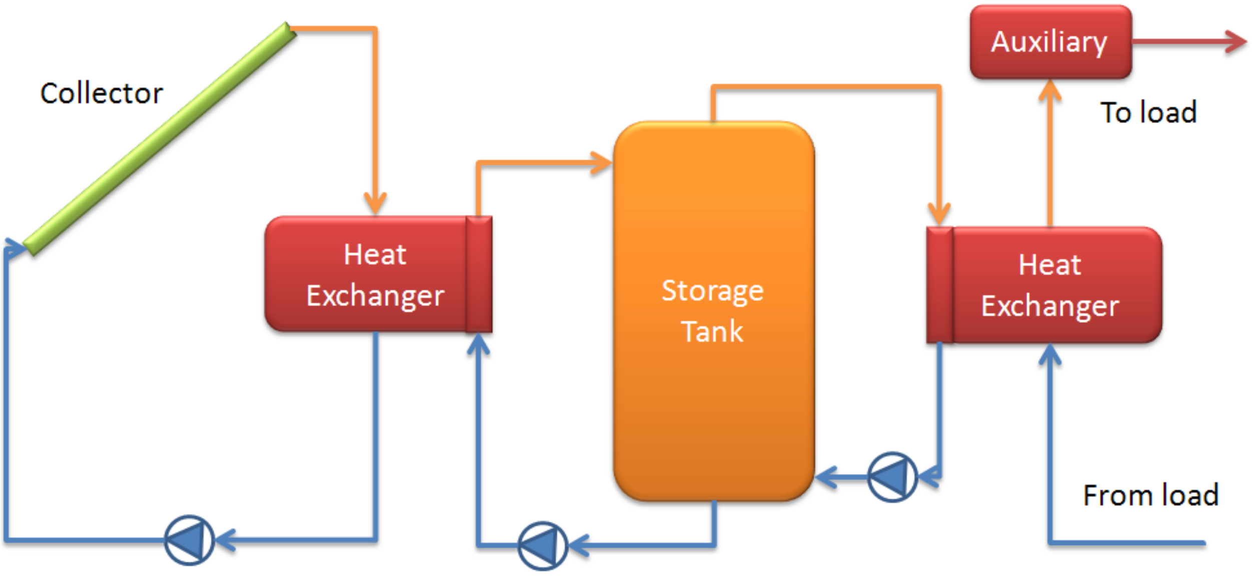

Figure 1: Schematic diagram of a closed loop solar energy system

This system is composed by a solar collector array designed to collect heat by absorbing sunlight. After, the heat is transferred to the storage tank through a heat exchanger which separates the fluid used in the solar collector from water contained in the storage tank. Then the heat is transferred to the load through a circuit with an auxiliary heater that possibly compensates to reach the minimum required temperature.

The formulation of the system of equations yields to a nonlinear system of 12 equations in 12 unknowns that is solved in Scilab using the function "***fsolve***".

The paper is organized as follows. First we present the system equations for the φ-f chart method and then we apply the system to an example.

2. The φ-f chart method

In this section we only report the formula used in the implementation of the φ-f chart method. For a more detailed description see references [1], [2] and [3].

2.1. Symbols

Table 1 refers to the problem unknowns while Table 2 reports all the used nomenclature.

Table 1: Problem unknowns

Variable	Description	Units
f	Solar fraction	[-]
f_{tl}	Total fraction of the monthly load supply by solar energy	[-]
L_{tot}	Total monthly heat demand	[J]
Q_s	Storage tank losses for month	[J]
T_i	Inlet collector fluid temperature	[°C]
$T_{p,min}$	Useful energy temperature	[°C]
$\overline{T}_s$	Monthly average storage tank temperature	[°C]
X	Modified sensibility factor of the thermal losses	[-]
$\overline{X}_c$	Dimensionless average daily critical level of the solar collector	[-]
$\overline{X}_{c,min}$	Minimum of the dimensionless average daily critical level of the solar collector	[-]
Y	Ratio of the absorbed solar energy to the cooling load	[-]
$\overline{\Phi}_{max}$	Maximum monthly average daily utilizability	[-]

Table 2: Nomenclature

Variable	Description	Units
a, b, c	Fitting constants of $\overline{\Phi}$ equation	
A_c	Collector area	[m^2]
c_p	Specific heat capacity	[J/kg K]
C_{min}	Minimum capacitance of the two fluid stream in the heat exchanger	[W/°C]

$\boldsymbol{F_R}$	Collector overall heat removal efficiency factor	[-]
$\boldsymbol{F'_R}$	Corrected F_R factor	[-]
$\boldsymbol{f}$	Solar fraction	[-]
$\boldsymbol{f_{tl}}$	Total fraction of the monthly load supply by solar energy	[-]
$\boldsymbol{\overline{H}}$	Monthly average daily total solar radiation on a horizontal surface	[Jm^{-2}]
$\boldsymbol{\overline{H}_0}$	Monthly average daily extraterrestrial solar radiation	[J/m^2]
$\boldsymbol{\overline{H}_t}$	Monthly average daily total solar radiation on the collector surface	[Jm^{-2}]
$\boldsymbol{\overline{K}_T}$	Clearness index	[-]
$\boldsymbol{L}$	Heat required	[J]
$\boldsymbol{L_{req}}$	Process required heat rate	[W]
$\boldsymbol{L_{tot}}$	Total monthly heat demand	[J]
$\boldsymbol{M}$	Actual mass of storage capacity	[kg]
$\boldsymbol{N}$	Numbers of days in a month	
$\boldsymbol{Q_s}$	Storage tank losses for month	[J]
$\boldsymbol{r_n}$	Ratio of the diffuse solar radiation at solar noon to the daily total radiation on a horizontal surface	[-]
$\boldsymbol{\overline{R}}$	Ratio of the monthly average daily total radiation on a tilted surface to that on a horizontal surface	[-]
$\boldsymbol{R_n}$	Ratio of radiation on a tilted surface to that on a horizontal surface at noon	[-]
$\boldsymbol{R_s}$	Ratio of standard storage heat capacity per unit of collector area of 350 kJ/m^2-°C to actual storage capacity	[-]
$\boldsymbol{\overline{T}_a}$	Ambient temperature	[°C]
$\boldsymbol{T_i}$	Inlet collector fluid temperature	[°C]
$\boldsymbol{T_{min}}$	Minimum required temperature	[°C]
$\boldsymbol{T_{p,min}}$	Useful energy temperature	[°C]
$\boldsymbol{\overline{T}_s}$	Monthly average storage tank temperature	[°C]
$\boldsymbol{T_{s,env}}$	Environment storage temperature	[°C]

$(UA)_s$	Overall loss coefficient-area product of the storage tank	[W/K]
U_L	Collector overall energy loss coefficient	[W/m^{2}K];
X	Modified sensibility factor of the thermal losses	[-]
$\bar{X}_c$	Dimensionless average daily critical level of the solar collector	[-]
$\bar{X}_{c,min}$	Minimum of the dimensionless average daily critical level of the solar collector	[-]
Y	Ratio of the absorbed solar energy to the cooling load	[-]
a_Φ, b_Φ, c_Φ, d_Φ	Constants fitting data of $\bar{\Phi}_{max}$ equation	
ε_L	Effectiveness of the load heat exchanger	[-]
Δt	Total number of seconds in a month	[s]
Δt_h	Number of seconds during a month the load is required	[s]
$\overline{(\tau\alpha)}$	Monthly average energy transmittance–absorptance product	[-]
$(\tau\alpha)_n$	Transmittance–absorptance product for radiation at normal incidence	[-]
$\bar{\Phi}$	Monthly average daily collector utilizability	[-]
$\bar{\Phi}_{max}$	Maximum monthly average daily utilizability	[-]

2.2. Equations 1 and 2

In the definition of the φ-f chart method it necessary to introduce two adimensional factors X and Y given by

$$X = \frac{A_c F_R' U_L (100)\Delta t}{L_{tot}} = \frac{A_c \frac{F_R'}{F_R} F_R U_L (100)\Delta t}{L_{tot}} \qquad (1)$$

$$\bar{\Phi}_{max} Y = \bar{\Phi}_{max} \frac{A_c F_R' \overline{(\tau\alpha)} \bar{H}_t N}{L_{tot}} = \bar{\Phi}_{max} \frac{A_c \frac{F_R'}{F_R} F_R (\tau\alpha)_n \frac{\overline{(\tau\alpha)}}{(\tau\alpha)_n} \bar{H} \bar{R} N}{L_{tot}} \qquad (2)$$

where

- $\bar{H}_t = \bar{H} \cdot \bar{R}$;
- $\Delta t = N \cdot 24 \cdot 3600$.

2.3. Equations 3 and 4

The total monthly heat demand L_{tot} is sum of the monthly heat user demand and the storage tank loss

$$L_{tot} = L + Q_s \qquad (3)$$

where the storage tank losses for month Q_s are given by the relation

$$Q_s = (UA)_s \cdot (\bar{T}_s - T_{s,env}) \cdot \Delta t \qquad (4)$$

The heat required is given by

$$L = L_{req} \cdot \Delta t_h$$

where

- $\Delta t_h = N \cdot hours \cdot 3600$;
- $hours = \frac{\text{hours daily} \cdot \text{number of working days}}{\text{number of days in one year}} = \frac{8 \cdot 250}{365}$

2.4. Equations 5

The $\bar{\Phi}_{max}$ term is related to the total fraction of the monthly load supplied by solar energy term f_{tl}, the relation is:

$$f_{tl} = \bar{\Phi}_{max} \cdot Y - a_{\Phi} \cdot (\exp(b_{\Phi} \cdot f_{tot}) - 1) \cdot (1 - \exp(c_{\Phi} \cdot X)) \cdot R_s^{d_{\Phi}} \qquad (5)$$

where

- R_s is given by the relation

$$R_s = \frac{350}{\frac{Mc_p}{A_c}}$$

$\frac{M}{A_c}$ is known as specific storage.

2.5. Equations 6 and 7

The average daily utilizability is given from the formula

$$\bar{\Phi} = \frac{f_{tl}}{Y} \tag{6}$$

where $\bar{\Phi}$ relates $\bar{\Phi}_{max}$ to the clearness index $\bar{K}_T$ by the relation

$$\bar{\Phi}_{max} = \bar{\Phi}(\bar{X}_{c,min}) \tag{7}$$

where:

$$\bar{\Phi}(\bar{X}_c) = \exp\left((a + bR_n/\bar{R}) \cdot \left(\bar{X}_c + c\bar{X}_c{}^2\right)\right)$$

$$a = +2.943 - 9.271 \cdot \bar{K}_T + 4.031 \cdot \bar{K}_T{}^2$$

$$b = -4.345 + 8.853 \cdot \bar{K}_T - 3.602 \cdot \bar{K}_T{}^2$$

$$c = -0.170 - 0.306 \cdot \bar{K}_T + 2.936 \cdot \bar{K}_T{}^2$$

All constants come from experimental data fitting as reported in [1, 2, and 3].

2.6. Equations 8 and 9

The $\bar{X}_c$ term is equal to

$$\bar{X}_c = \frac{I_{tc}}{r_n R_n \bar{H}} = \frac{\frac{F_R U_L (T_i - \bar{T}_a)}{F_R \overline{(\tau\alpha)}}}{r_n R_n \bar{H}_0 \bar{K}_T} \tag{8}$$

where

The $\bar{X}_{c,min}$ is the minimum of the dimensionless average daily critical level of the solar collector $\bar{X}_c$ which can be computed as

$$\bar{X}_{c,min} = \frac{\frac{F_R U_L \left(T_{p,min} - \bar{T}_a\right)}{F_R (\tau\alpha)_n \frac{\overline{(\tau\alpha)}}{(\tau\alpha)_n}}}{r_n R_n \bar{H}} \tag{9}$$

2.1. Equations 10

For the temperature in the storage tank $\bar{T}_s$ we have the following equation:

$$\bar{T}_s = \frac{T_{p,min} + T_i}{2} \tag{10}$$

2.2. Equations 11

The heat exchanger adds a resistive between the storage tank and the load. Doing so, it reduces the useful energy and increase storage tank losses. This can be modeled by the following formula:

$$T_{p,min} - T_{min} = \frac{fL\,/\Delta t_h}{\varepsilon_L C_{min}} \tag{11}$$

2.3. Equations 12

The solar fraction f can be related to the total fraction of the monthly load supplied by solar energy f_{tl} and to the useful load by the relation:

$$f = f_{tl} \cdot \left(1 + \frac{Q_s}{L}\right) - \frac{Q_s}{L} \tag{12}$$

2.4. Expressions

The above formula requires the evaluation of some solar constants such as R_n, $\bar{H}$. This is done starting from the data where the plant is located like latitude, tilt angle of solar collector, etc. See [1], [2] and [3] for further details.

3. Example

An industrial process heat system has a 100 m^2 collector. The system is located in Rome (41° 53′ 35″ N latitude), and the collector characteristics are $F_R U_L = 4.5$ W/ m^2-°C, $F_R(\tau\alpha)_n = 0.85$, $F_R'/F_R = 0.95$ tilted at 40° (south orientation). The process requires heat at a rate of 20 kW at a temperature of 70 °C for 8 h each day. The Specific storage is 120 kg m^{-2} of water and the tank has an overall loss coefficient $(UA)_s = 14$ WK^{-1}. The heat exchanger has effectiveness $\varepsilon_L = 0.75$ and $C_{min} = 5000$ WK^{-1}.

Results are reported in the following tables and figures.

Table 3: Problem data

month	H_mean	K_t	R_mean	R_n	tau_alpha	T_a [°C]	L [J]
1	5976.59	0.428	1.6281	1.4930	0.9029	8.8	1.2230E+10
2	8286.08	0.428	1.3791	1.3193	0.9039	10.3	1.1047E+10
3	11951.50	0.451	1.1802	1.1844	0.9031	13.1	1.2230E+10
4	16169.56	0.474	1.0155	1.0728	0.8970	16.4	1.1836E+10
5	19940.26	0.504	0.9128	1.0007	0.8904	20.8	1.2230E+10
6	21603.19	0.518	0.8688	0.9688	0.8863	24.8	1.1836E+10
7	22233.37	0.553	0.8867	0.9883	0.8934	28.0	1.2230E+10
8	19434.27	0.548	0.9778	1.0557	0.9026	27.8	1.2230E+10
9	14837.85	0.522	1.1366	1.1640	0.9094	24.3	1.1836E+10
10	10527.86	0.507	1.3741	1.3222	0.9127	18.8	1.2230E+10
11	6223.64	0.421	1.5436	1.4337	0.9028	13.9	1.1836E+10
12	4708.55	0.385	1.6369	1.4995	0.8976	9.8	1.2230E+10

Table 4: Numerical results

month	X_c	X_c_min	X	Y	phi_max	T_i [°C]	T_p_min [°C]	T_s [°C]	L_tot [J/month]	Q_s [J]	f	f_tot
1	0.865	0.835	8.064	1.549	0.246	73.625	71.367	72.496	1.420E+10	1.968E+09	0.256	0.359
2	0.781	0.742	8.039	1.815	0.282	75.330	72.068	73.699	1.287E+10	1.819E+09	0.388	0.474
3	0.677	0.619	7.990	2.225	0.333	78.836	73.188	76.012	1.433E+10	2.100E+09	0.598	0.657
4	0.617	0.538	7.943	2.558	0.370	82.495	74.057	78.276	1.395E+10	2.115E+09	0.761	0.797
5	0.574	0.475	7.899	2.799	0.406	86.047	74.744	80.395	1.449E+10	2.265E+09	0.889	0.907
6	0.552	0.439	7.936	2.886	0.433	88.110	75.096	81.603	1.396E+10	2.127E+09	0.956	0.962
7	0.527	0.387	7.943	3.058	0.479	92.868	75.665	84.267	1.441E+10	2.185E+09	1.062	1.053
8	0.526	0.387	7.925	2.971	0.484	92.736	75.603	84.170	1.445E+10	2.219E+09	1.051	1.043
9	0.549	0.441	7.927	2.658	0.445	87.257	74.887	81.072	1.398E+10	2.144E+09	0.916	0.929
10	0.602	0.532	7.962	2.298	0.381	80.989	73.777	77.383	1.438E+10	2.152E+09	0.708	0.752
11	0.823	0.791	8.059	1.528	0.269	73.924	71.531	72.727	1.375E+10	1.913E+09	0.287	0.386
12	1.026	1.000	8.087	1.223	0.206	72.198	70.647	71.423	1.416E+10	1.928E+09	0.121	0.241

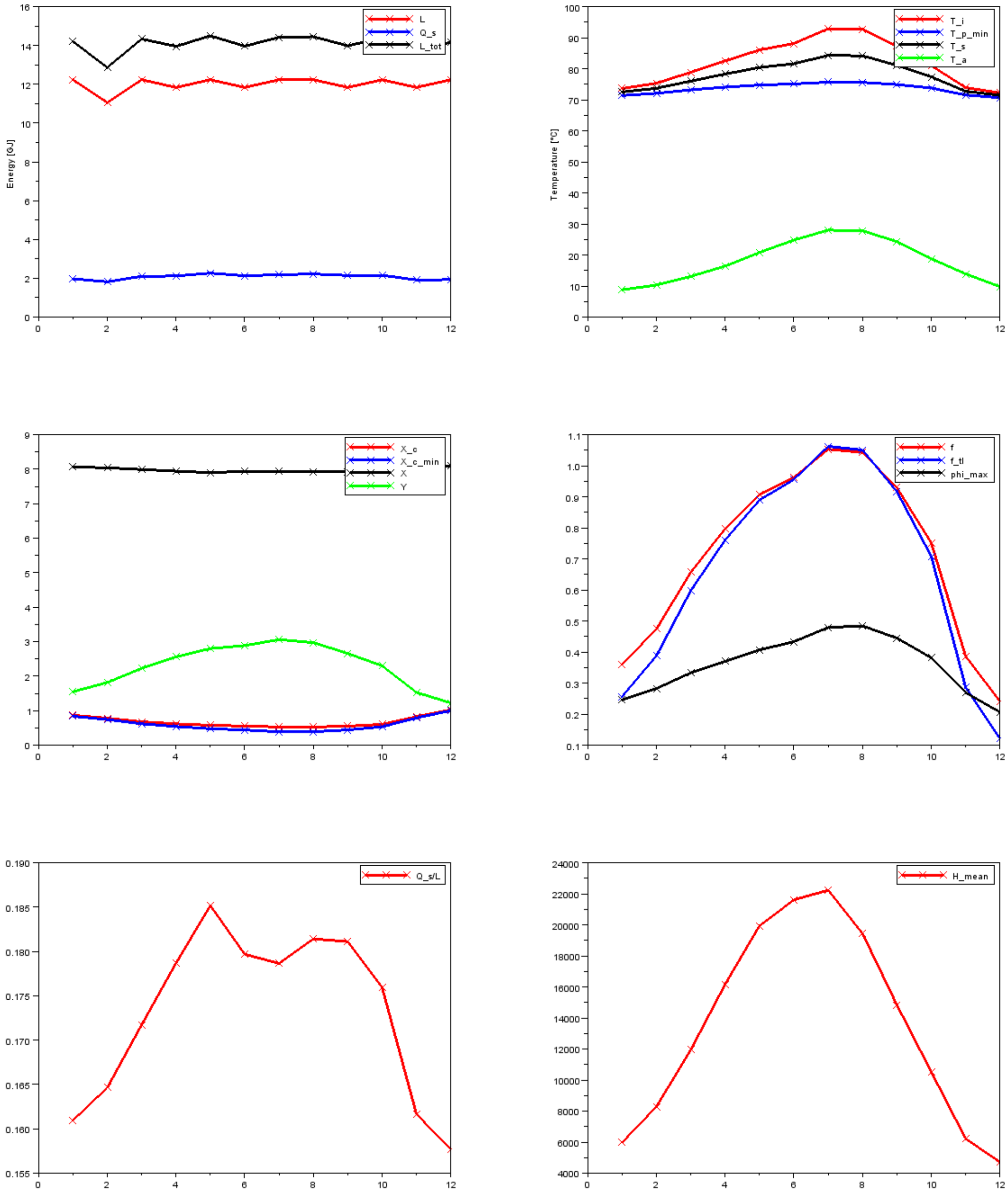

Figure 2: Numerical results during the 12 months. From top to bottom: energies, temperatures, factors, fractions, Q_s/L and H_mean.

4. Varying the parameters

In this section we provide some results about parameters variation. The previous example is here simulated by varying one parameter at once. The following figures report the solar fraction for hot water production during the year.

Figure 3 shows the solar fraction during the year for three different collecting areas: 80 m^2, 90 m^2, 100 m^2. It is evident that increasing the collecting area, the faction increases. Moreover we may notice that the highest fraction in Rome is reached in July. This is due to the maximum value of the incident solar radiation (see H_mean chart in the previous section).

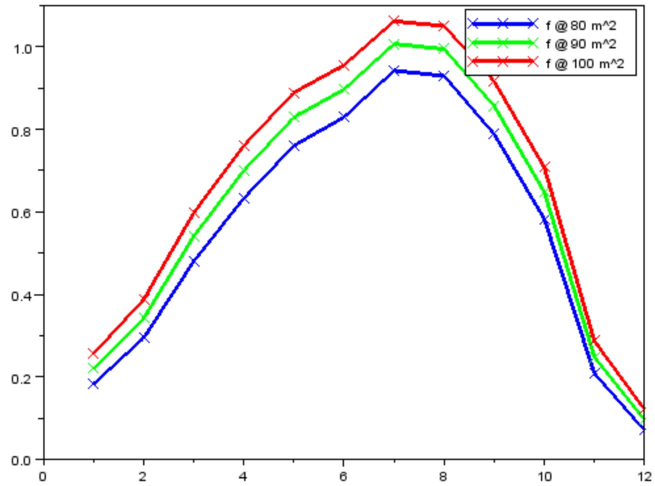

Figure 3: Solar fraction with different collecting areas.

Figure 4 describes the behavior of the solar fraction during the year for three different tilt angles of the solar collector: 20°, 30°, and 40°. It is visible that configurations with lower angles are better during the summer and higher values of angles are better during the winter. This is due to the sun’s apparent position during the year. In few words, this means that a fixed tilt angle cannot be optimal for all the months and that an automatic change in the inclination could be advisable.

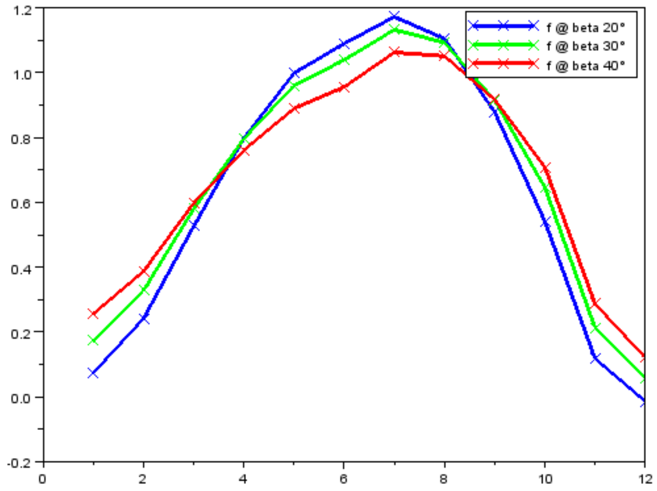

Figure 4: Solar fraction with three different tilt angles of solar collector.

Figure 5 shows the behavior of the solar fraction during the year for three different specific storages: 80 kg/m^2, 100 kg/m^2, 120 kg/m^2. This behavior is very simple, the more the mass, the more the fraction. The effect is more visible during the summer than during the winter.

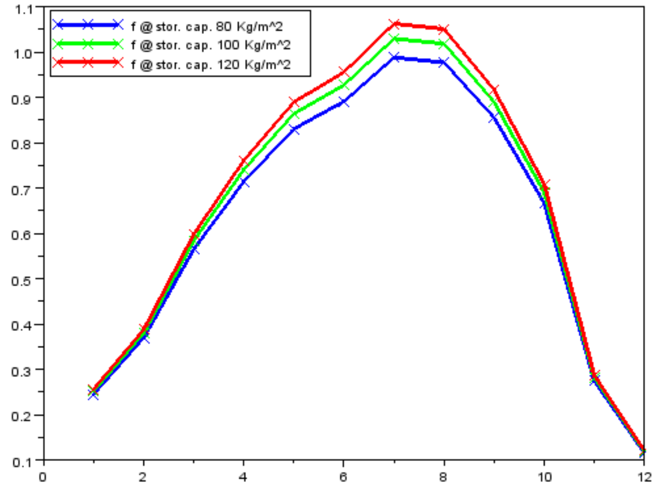

Figure 5: Solar fraction with different specific storages.

Figure 5 contains the last set of experiments that have been carried out by changing the minimum required temperature. The experiments have been done for three different values: 60°C, 65°C, and 70°C. The effect is simple all year round, the less the requirement, the higher the solar fraction.

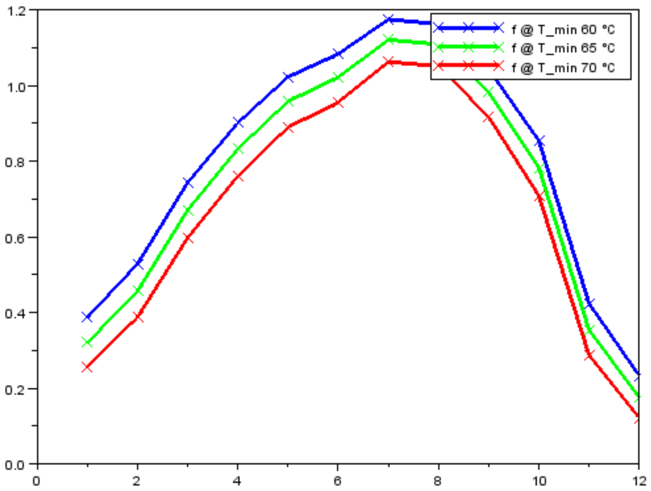

Figure 6: Solar fraction with different minimum temperature requirements.

5. Conclusion

We developed a Scilab module for the study of solar energy systems using the well-known φ-f chart method. This model is completely parameterized and very fast. The advantage of having such a kind of model is that it may return within few seconds all relevant system quantities necessary to calibrate an optimal system.

The presented model is intended to be integrated with costs, allowing engineers to design the most appropriate solar energy systems at the minimum cost. This can be done directly within Scilab using one of the available optimization algorithms [4].

6. References

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